DICK SCHAFF MATH SUPERBOWL XLVI
Level 5 Blitz: Secondary Math III - 2019

Directions: 1) Select the most correct answer for each question and mark it on your answer form.
2) No calculators of any sort are allowed.
3) Note the N.O.T. means "None of these."

#1. The vertex of the parabola $y = \frac{1}{2}x^2 + 4x + 4$ is:

a) $(-4, -4)$ b) $(-4, 0)$ c) $(-1, \frac{7}{2})$ d) $(-1, \frac{9}{2})$ e) N.O.T.

#2. The remainder when $2x^3 + 3x^2 - 2x + 3$ is divided by $x + ½$ is:

a) 1.5 b) 4.5 c) 0.5 d) -0.5 e) N.O.T.

#3. When $4x^3 - 2x^2 + 2x + 5$ is divide by $2x - 1$, the remainder is:

a) -7 b) -3 c) 4 d) 6 e) N.O.T.

#4. Find the solution set to the radical equation $\sqrt{3x + 1} = 2x$.

a) $\{1, -¼\}$ b) $\{1, -1\}$ c) $\{1\}$ d) $\{0, 1\}$ e) N.O.T.

#5. If s_1 and s_2 are the solutions to the quadratic equation $2x^2 - 4x + 3 = 0$, then $s_1 s_2$ is:

a) $\frac{3}{2}$ b) 2 c) 10 d) 20 e) N.O.T.

#6. What is the coefficient of the x^3y^2 term in the binomial expansion of $(x - 3y)^5$?

a) 90 b) 30 c) 15 d) 10 e) N.O.T.

#7. The legs of a right triangle are in a ratio of 3 to 1. If θ is the smallest of the two acute angles, then $\csc\theta$ is:

a) $\frac{\sqrt{10}}{10}$ b) $\frac{3\sqrt{10}}{10}$ c) $\frac{\sqrt{10}}{3}$ d) $\sqrt{10}$ e) N.O.T.

#8. Find the solution set to the rational equation $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{(x-4)(x-2)}$.

a) $\{4,-1\}$ b) $\{-2\}$ c) No Solution d) $\{-1\}$ e) N.O.T.

#9. If $f(x)=\frac{x+1}{x-1}$, then $f^{-1}(x)$ is:

a) $\frac{x-1}{x+1}$ b) $\frac{x+1}{x-1}$ c) $\frac{1-x}{1+x}$ d) $\frac{x+1}{1-x}$ e) N.O.T.

#10. In a set of test scores that are normally distributed, a test score of 38 is two standard deviations below the mean. A score of 83 is one standard deviation above the mean. What is the mean of the data?

a) 15 b) 60.5 c) 53 d) 68 e) N.O.T.

#11. If $f(x)=-2^{-x}$, then $f^{-1}(x)$ is:

a) $\log_2(-x)$ b) $\log_2(x)$ c) $\log_{\frac{1}{2}}(-x)$ d) $\log_{\frac{1}{2}}(x)$ e) N.O.T.

#12. How many real solutions does the equation $\log_3(x+2)+\log_3(x-2)=1+\log_3(x)$ have ?

a) 0 b) 1 c) 2 d) 3 e) N.O.T.

#13. The set of values for k for which the quadratic equation $kx^2+2x+k=0$ has real solutions is:

a) $(-\infty,-1]\cup[1,\infty)$ b) $[-1,1]$ c) $(-\infty,\infty)$ d) ϕ e) N.O.T.

#14. An arc is measured counterclockwise around a circle centered at the origin from the positive x - axis to the point $\left(-\frac{\sqrt{3}}{2},\frac{3}{2}\right)$. The measure of this arc is:

a) $\frac{2\pi}{\sqrt{3}}$ b) $2\pi\sqrt{3}$ c) $\frac{5\pi}{2\sqrt{3}}$ d) $\frac{5\pi}{6\sqrt{3}}$ e) N.O.T.

For questions 15 – 18, consider the rational function $f(x) = \frac{x^2-3x-4}{x^2-x-2}$.

#15. Which of the following is an x-intercept of f(x)?

a) $(-1, 0)$ b) $(2, 0)$ c) $(0, 2)$ d) $(0, 4)$ e) N.O.T.

#16. Which of the following is a y-intercept of f(x)?

a) $(-1, 0)$ b) $(2, 0)$ c) $(0, 2)$ d) $(0, 4)$ e) N.O.T.

#17. Which of the following is a vertical asymptote of f(x)?

a) $x = -1$ b) $x = 2$ c) $y = 0$ d) $y = 1$ e) N.O.T.

#18. Which of the following is a horizontal asymptote of f(x)?

a) $x = -1$ b) $x = 2$ c) $y = 0$ d) $y = 1$ e) N.O.T.

#19. A line perpendicular to $x + 3y = 4$ contains the point (1, 6). If the perpendicular intersects $x + 3y = 4$ at the point (a, b), then $a + b$ is:

a) -3 b) 0 c) 1 d) 4 e) N.O.T.

#20. Which of the following is the inverse of $f(x) = 1.5^x + 4$?

a) $f^{-1}(x) = \frac{x-4}{1.5}$ b) $f^{-1}(x) = \frac{\log(x)-4}{1.5}$

c) $f^{-1}(x) = \frac{\log(x-4)}{\log(1.5)}$ d) $f^{-1}(x) = \frac{4-\log(x)}{\log(1.5)}$ e) N.O.T.

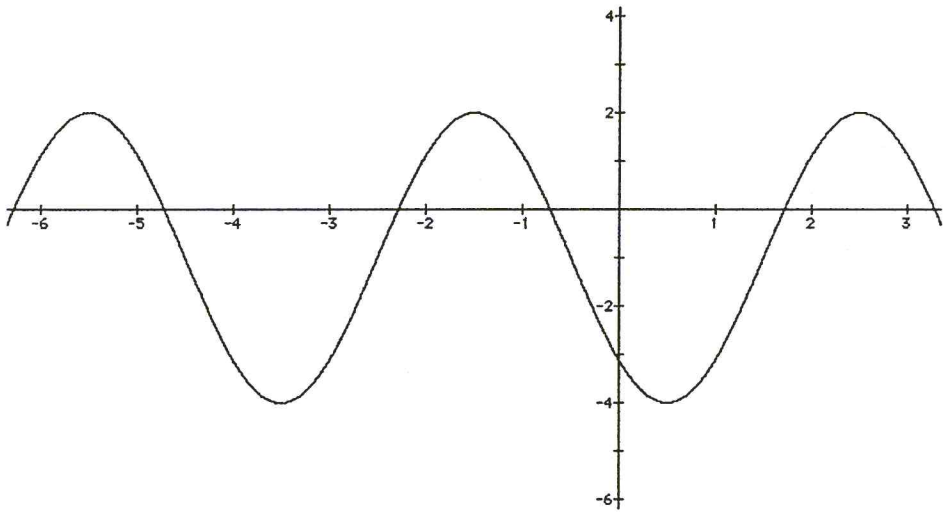
#21. If $\frac{3}{z} = \frac{6}{x} - \frac{2}{y}$, then x is:

a) $3y+2z$ b) $\frac{2yz}{y+2z}$ c) $\frac{yz}{y+z}$ d) $\frac{6yz}{3y+2z}$ e) N.O.T.

#22. Find the solution set to the exponential equation $e^{5x} = 2019$.

a) $\frac{5}{\ln(2019)}$ b) $\frac{\ln(2019)}{5}$ c) $\frac{\ln(5)}{2019}$ d) $\frac{2019}{\ln(5)}$ e) N.O.T.

#23. If the equation of the graph shown below is in the form $y = A\sin(\omega x + \phi) + k$, then the value of ω is:



a) $\frac{\pi}{2}$ b) π c) $\frac{1}{2}$ d) 2 e) N.O.T.

#24. i^{2019} is

a) i b) -1 c) $-i$ d) 1 e) N.O.T.

#25. The value of $\sin 71°\cos(-19°) - \sin(-19°)\cos 71°$ is:

a) -1 b) 0 c) $\frac{1}{2}$ d) 1 e) N.O.T.

#26. The value of $log_2(log_5 125)$ is

a) 1 b) 2 c) 3 d) 8 e) N.O.T.

#27. For which set of numbers is the function $f(x) = \ln\left(\frac{x}{x^2-1}\right)$ defined ?

a) $(-\infty,-1)$ b) $(-1,0)$ c) $(0,1)$ d) $(-1,1)$ e) N.O.T.

#28. The domain of the function $f(x) = -\ln(-x+2)$

a) $(-\infty,-2)$ b) $(-\infty,-2]$

c) $(-\infty,\ 2)$ d) $(-\infty,\ 2]$ e) N.O.T.

#29. An asymptote of the hyperbola $4x^2 - y^2 - 4y = 0$ is:

a) $y = 2x-2$ b) $y = -2x+2$

c) $y = \frac{1}{2}x-2$ d) $y = -\frac{1}{2}x+2$ e) N.O.T.

#30. The y-intercept of the line that is perpendicular to the line with equation $x + 2y = 4$ and passing through the point $(-2,\ 1)$ is

a) $(0,5)$ b) $(0,-5)$ c) $(0,0)$ d) $(0,\frac{1}{5})$ e) N.O.T.

#31. The maximum value of the function $f(x) = \sqrt{6x-x^2}$ is:

a) 0 b) 3 c) 6 d) 9 e) N.O.T.

#32. The sum of the series $\sum_{n=1}^{100}(6-\frac{1}{2}n)$ is

a) 100 b) 225 c) 1925 d) -1925 e) N.O.T.

#33. The vertex of the parabola $y = ax^2+bx+c$ is (h,k). If the point $(h-4,k+4)$ is on the parabola, then the value of a is:

a) $\frac{1}{16}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) $\frac{1}{2}$ e) N.O.T.

#34. The coefficient of x^2 in the expansion of $\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)^8$ is

a) 28 b) 252 c) 1,512 d) 5,670 e) N.O.T.

#35. If the graph of $f(t) = P_0 e^{kt}$ contains the points $(5,5)$ and $(10,10)$, then the value of k is:

a) $\ln\frac{5}{2}$ b) $\frac{1}{5}\ln 2$ c) $\frac{1}{2}\ln 5$ d) $2\ln 5$ e) N.O.T.

#36. Find the solution set to the equation $\sqrt{3}sin\theta + cos\theta = 1$ on the interval $0 \leq \theta < 2\pi$.

a) $\left\{0, \frac{2\pi}{3}\right\}$ b) $\left\{\frac{\pi}{4}\right\}$ c) $\left\{0, \frac{5\pi}{3}\right\}$ d) $\left\{0, \frac{7\pi}{4}\right\}$ e) N.O.T.

#37. The domain of $f(x) = 2\sec\left(\frac{1}{2}x - \frac{\pi}{4}\right)$ is: (k is an integer)

a) $\left\{x \middle| x \neq \frac{\pi}{2} \pm 2\pi k\right\}$ b) $\left\{x \middle| x \neq \frac{\pi}{2} \pm 4\pi k\right\}$

c) $\left\{x \middle| x \neq \frac{3\pi}{2} \pm 2\pi k\right\}$ d) $\left\{x \middle| x \neq \frac{3\pi}{2} \pm 4\pi k\right\}$ e) N.O.T.

#38. If $\cot\beta = \frac{\sqrt{2}}{2}$, then $\tan 2\beta$ is equal to

a) $-\sqrt{2}$ b) $\sqrt{2}$ c) $-2\sqrt{2}$ d) $2\sqrt{2}$ e) N.O.T.

#39. The solution to the equation $3^{x+3} = 6^x$ is:

a) $x = \frac{3}{1-\ln 2}$ b) $x = \frac{3\ln 3}{\ln 2}$

c) $x = \frac{3\ln 3}{\ln 6 - \ln 3}$ d) $x = \frac{\ln 9}{\ln 6 - \ln 3}$ e) N.O.T.

#40. Simplify $log_2(3) \times log_3(4) \times log_4(5) \times \ldots \times log_{2018}(2019)$.

a) log(2019) b) ln(2019)

c) log(1009.5) d) ln(1009.5) e) N.O.T.

#41. The graph of $f(x) = -3 + \sqrt{x^2 - 9}$ is a portion of:

a) a circle b) an ellipse

c) a parabola d) a hyperbola e) N.O.T.

#42. Find the solution set for the logarithmic equation $\log(x+1) - \log(x+3) = \log\left(\frac{1}{x}\right)$.

a) $\{1\}$ b) $\{\sqrt{3}\}$ c) $\{3\}$ d) $\{\pm\sqrt{3}\}$ e) N.O.T.

#43. If $f(x) = \frac{1}{x+1}$, then $\frac{f(x+h) - f(x)}{h}$ is:

a) $\frac{1}{(x+1)(x+h+1)}$ b) $\frac{-1}{(x+1)(x+h+1)}$

c) $\frac{1}{(x+h+1)^2}$ d) $\frac{-1}{(x+h+1)^2}$ e) N.O.T.

#44. Solve for x in the following: $\frac{c}{x} + 1 = b, with\ b > 1$

a) $\frac{c}{b-1}$ b) $\frac{c}{1-b}$ c) $\frac{c}{b+1}$ d) $\frac{c}{b} - c$ e) N.O.T.

#45. The angle of elevation (line of sight) from an observation point on the ground to the top of a tree is 45°. The angle of elevation to a point 12 feet below the top of the tree is 30°. The height of the tree (in feet) is:

a) $6\left(3-\sqrt{3}\right)$ b) $6\left(3+\sqrt{3}\right)$

c) $12\left(1+\sqrt{3}\right)$ d) $12\left(3-\sqrt{3}\right)$ e) N.O.T.